Relaxation Impedance – One Reason for Inductive and Capacitive Behavior in Low Frequency Impedance Spectra of Corroding Electrodes, Batteries and Fuel Cells

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Overview

- Electrochemical systems may exhibit significant inductive contributions to the overall frequency response appearing in particular at low frequencies, when an Ohmic course is expected.

- Also the complementary behavior may occur: Expected is a monotonous Ohmic course of the low frequency impedance, but instead an additional capacitive loop can be observed.

- Many different mechanisms are discussed in the literature explaining special situations, which are leading to inductive or capacitive behavior of electrochemical systems in the low frequency range.

- A simple mechanism may give a common explanation for a lot of such appearances. It traces back such unexpected reactance contributions to the time dependency of some mediating physical magnitude, affecting the impedance.

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Example* EIS at Polymer Fuel Cells (PEFC): Contributions to the overall U-i characteristic can be determined by EIS

How Do We Model the Faraday Impedance?

Dynamic transfer function $Z_F$ between AC current $\tilde{I}_F$ and AC voltage $\tilde{U}$

$$Z_F = \frac{\tilde{U}}{\tilde{I}_F}, \quad \tilde{I}_F = f(\tilde{A}, \tilde{B}, \tilde{C}, \ldots; \ a, b, c, \ldots)$$

Modulated magnitudes: $\tilde{U}, \tilde{A}, \tilde{B}, \tilde{C}, \ldots$

“state variables”

$$\tilde{I}_F = \left(\frac{\partial \tilde{I}_F}{\partial \tilde{U}}\right) \cdot \tilde{U} + \left(\frac{\partial \tilde{I}_F}{\partial \tilde{A}}\right) \cdot \tilde{A} + \left(\frac{\partial \tilde{I}_F}{\partial \tilde{B}}\right) \cdot \tilde{B} + \left(\frac{\partial \tilde{I}_F}{\partial \tilde{C}}\right) \cdot \tilde{C} + \ldots$$

where $\tilde{U}, \tilde{I}_F, \tilde{A}, \tilde{B}, \tilde{C}, \ldots$ denote the non-periodic shares of the magnitudes.

Time independent magnitudes: $a, b, c, \ldots$

“kinetic parameters”

The Charge Transfer Resistance $R_\eta = \frac{\partial \tilde{U}}{\partial \tilde{I}_F}$: One important contribution to the total Faraday Impedance.

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Butler-Volmer Equation:

\[ i = i_0 \cdot \left[ \exp(\alpha \cdot z \cdot F \cdot \eta) - \exp\left( -(1-\alpha) \cdot z \cdot F \cdot \eta \right) \right] \]

Deduction:

\[ i^+ = z \cdot F \cdot k^+ \cdot c_A^a \cdot \exp\left( -\frac{W^+}{RT} \right), \quad z > 0, \]

\[ i^- = z \cdot F \cdot k^- \cdot c_B^b \cdot \exp\left( -\frac{W^-}{RT} \right), \quad z < 0, \]

\[ i = i^+ + i^- \]

\[ k^+, k^- : scaling factors (reaction rate constants) \]

\[ W^+, W^- : activation energy (anod., cathod.) \]

Remarkable simplification: only \( W^+, W^- \) depend on the potential!

\[ W^+ = W_0^+ - \alpha \cdot z \cdot F \cdot (\varepsilon_r - \varepsilon^0 + \eta) \]

\[ W^- = W_0^- + (1-\alpha) \cdot z \cdot F \cdot (\varepsilon_r - \varepsilon^0 + \eta) \]

\[ \varepsilon_r : reversible reference potential \]

\[ \varepsilon^0 : reversible standard reference potential \]

\[ \eta : charge transfer over-potential \]

Result from the Butler-Volmer Equation

Charge-transfer resistance \( R_\eta \) under the assumption that only the effective activation energies \((W^+, W^-)\) depend on the potential:

\[
\frac{1}{R_\eta} = \frac{\partial i}{\partial \eta} = \left| \frac{z}{R T} F \right| (\alpha \cdot i^+ + (1 - \alpha) \cdot |i^-|)
\]

\[
\frac{1}{R_\eta} = \left| \frac{z}{R T} F \cdot i_0 \right|
\]

at equilibrium

What happens if, for instance, the rate constants \( k^+, k^- \) or the symmetry coefficient \( \alpha \) would depend on the potential?
Long Time Scale Variation on Potential or Current - Approach: Relaxation with Time*

- Relaxation is assumed as a simple process, resetting a small perturbation ($\Delta$) of a physical quantity ($X$) with influence on the impedance after a potential or current change according to a first order rate equation involving just one time constant ($\tau$).

$$- \frac{dX_t}{dt} = \frac{1}{\tau} \left( X_t - \bar{X} \right) \quad \text{with} \quad \bar{X} = \text{stationary value}$$

- Consequently $X$ must be considered as a modulated state variable instead as of a steady state kinetic parameter.

$$\bar{X} = \hat{X} \cdot e^{j\omega t} \Rightarrow j \omega \bar{X} = -\frac{1}{\tau} \left( \bar{X} - \frac{dX}{dU} \cdot \bar{U} \right)$$

for $X \equiv k(U) \Rightarrow - \frac{dk_t}{dt} = \frac{1}{\tau_k} \left( \Delta k - \frac{d\bar{k}}{dU} \Delta U \right) \quad \text{with} \quad k = \text{reaction rate const.}$

Finding from 1986*

\[ Z_F = R_\eta + Z_c + Z_\theta \]

\[ -\frac{dk_t}{dt} = \frac{1}{\tau} \cdot (k_t - k) \]

\[ Z_F = \frac{R_\eta + Z_c + Z_\theta}{1 + R_\eta / Z_k} \]

\[ Z_k = \frac{1 + j\omega\tau}{I_F \cdot d \ln k / d\varepsilon} \]

Mild steel Fe_{38}Mn_{6} in aqueous chloride containing electrolyte in presence of CO\textsubscript{2}. Potential was around -0.5V vs. SHE. The pink curve in the left diagram is the ZHIT transform of the experimental phase angle data (red curve).

Some Experimental Results for $k=k(U)$ (2001)∗

23cm$^2$-PECF-single cell at a current density of 217mA/cm$^2$. Left: Spectra series (complex plane representation) after starting an aimed poisoning of the anode by adding 100ppm CO to the fuel. Right: One spectrum (Bode representation) after 4h.

Some Recent Result*

Time dependency of impedance spectra measured at -700 mV vs. Hg/HgO, silver membrane with 1.2 µm pore diameter

\[ X = X(U, I) \]

- Up till now only the influence of the voltage on kinetic parameters is considered in the examples eg. \( k = k(U) \).
- What will happen, if contributions to the impedance on long time scales are influenced in addition by the current?
- Inspiration from literature (H. Landes and A. Hahn 1998): investigations of temperature effects within a high temperature fuel cell (SOFC)
Some Experimental Results for $X=X(U, I)$, $X=T$ (1998)

Impedance spectra series recorded at an SOFC-MEA at 950°C under different DC load (H. Landes & A. Hahn, thesis of A. Hahn (1998))

\[ Z_{real} = \frac{Z_{ideal} - \nu \cdot U}{1 + \nu \cdot I} \]

\[ v = \frac{\tanh \left( d \cdot \sqrt{\frac{c \cdot \rho}{\lambda}} j\omega \right)}{2 \cdot \lambda \cdot \sqrt{\frac{c \cdot \rho}{\lambda}} j\omega} \cdot \frac{dU}{dT} \]

Modeling the thermal flow in a fuel cell in order to estimate the temperature relaxation of the electrolyte resistance

- \( d \): heat penetration depth
- \( \lambda \): thermal conductivity
- \( c \): specific heat capacity
- \( \rho \): density of heat conducting medium

General Impedance Expressions for the Relaxation of $X(U, I)$

$$Z_{R,X} = \frac{R + \left( \frac{\partial P}{\partial I} \right)_U \cdot \left( \frac{\partial U}{\partial X} \right)_I \cdot \frac{dX}{dP} \cdot \nu}{1 - \left( \frac{\partial P}{\partial U} \right)_I \cdot \left( \frac{\partial U}{\partial X} \right)_I \cdot \frac{dX}{dP} \cdot \nu}$$

$$R = \left( \frac{\partial U}{\partial I} \right)_X$$

$$\nu = (1 + j \cdot \omega \cdot \tau)^{-1}$$

$$P = P(U, I)$$

If $X$ represents the temperature $T$ and $P$ the electrical power $N$...

$$R = \left( \frac{\partial U}{\partial I} \right)_T = \frac{U_T}{I_T}, \quad R = R_{HF} \quad and \quad V_c = \left( \frac{\partial N}{\partial U} \right)_I \cdot \left( \frac{\partial U}{\partial T} \right)_I \cdot \frac{dT}{dN}$$

...one obtains the expression for $Z_{R,T}$ which has to be verified

$$Z_{R,T} = \frac{R + R \cdot V_C \cdot \nu}{1 - V_C \cdot \nu}$$
Special Impedance Expression for the Relaxation of $T(U, I)$

$$Z_{R,T} = \frac{R + R \cdot V_C \cdot \nu}{1 - V_C \cdot \nu} \quad \nu = (1 + j \omega \tau)^{-1}$$

- $V_C$: dimensionless variation coefficient
- The expression influences the shape of the impedance in all 4 quadrants of the complex plane
**The Influence of $V_C$ on the Impedance**

<table>
<thead>
<tr>
<th>Parameter $V_C$</th>
<th>Low frequency limit $Z_0 = Z_R (\omega \rightarrow 0)$</th>
<th>Low frequency impedance course</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_C = 0$</td>
<td>$Z_0 = Z_\infty = R_{HF}$</td>
<td>no reactance contribution</td>
</tr>
<tr>
<td>$1 &gt; V_C &gt; 0$</td>
<td>$Z_0 &gt; Z_\infty$</td>
<td>capacitive around $\omega \approx 1/\tau$</td>
</tr>
<tr>
<td>$-1 &lt; V_C &lt; 0$</td>
<td>$Z_0 &lt; Z_\infty$</td>
<td>inductive around $\omega \approx 1/\tau$</td>
</tr>
<tr>
<td>$V_C &gt; 1$</td>
<td>$Z_0 &gt; Z_\infty, Z'_0 &lt; 0$ !</td>
<td>capacitive around $\omega \approx 1/\tau$</td>
</tr>
<tr>
<td>$V_C &lt; -1$</td>
<td>$Z_0 &lt; Z_\infty, Z'_0 &lt; 0$ !</td>
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<tr>
<td>$V_C = 1$</td>
<td>$Z_0 \rightarrow \infty$</td>
<td>pure capacitive for $\omega &lt;&lt; 1/\tau$</td>
</tr>
<tr>
<td>$V_C = -1$</td>
<td>$Z_0 \rightarrow 0$</td>
<td>pure inductive for $\omega &lt;&lt; 1/\tau$</td>
</tr>
</tbody>
</table>
What is a Suitable Reference System for Testing?

- Electrochemical systems like corroding electrodes, batteries, fuel cells exhibit partial aspects i.e. dependence $X(U)$ or $X(I)$ and strong effects appear only in strongly non-stationary systems.
- Temperature dependent resistors can act as suitable reference systems for verifying the theory experimentally in the full parameter range of $V_C$.
- Reference low frequency EIS measurements on NTC, PTC and light bulbs were performed. They exhibited all expected variations of the relaxation impedance appearance.
- The spectra could successfully modeled by means of the Relaxation Impedance Model.
Fitting of the experimental results using the General Relaxation Model applied on Temperature Relaxation

Impedance spectra series of an NTC (left) and a PTC (right) at different dc bias values. Experimental samples are depicted as symbols, fitting results as solid lines.
Summary

• A transfer-function for the comprehensive relaxation impedance was established and verified quantitatively using some reference systems.
• It includes previously found results as special cases.

Conclusions

• Care must be taken in EIS modeling regarding the assignment of impedance relevant magnitudes as modulated “state variables” or as stationary “kinetic parameters” – relaxation processes may be present.
• Neglecting relaxation effects may result in erroneous conclusions, deducing DC-data from AC-experiments.
Conclusion

Relevant frequency range for the detection of for instance

• Polarization resistance in fields of corrosion
• Charge transfer resistance of battery and fuel cell electrodes
• Gas diffusion limit resistance of a fuel cell anode
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Relaxation contribution in the case of a passivation process
Conclusion

Relevant frequency range for the detection of for instance

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Relaxation contribution in the case of a passivation process

Relaxation contribution in the case of an activation process
Thank you for your attention! Consider relaxation...

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\[ \tilde{X} = \hat{X} \cdot e^{j \omega t} \rightarrow j \omega \tilde{X} = -\frac{1}{\tau} \left( \tilde{X} - \frac{dX}{dU} \cdot \tilde{U} \right) \]

for \( X \equiv k(U) \) \( \Rightarrow -\frac{dk}{dt} = \frac{1}{\tau_k} \left( \Delta k - \frac{d\tilde{k}}{dU} \Delta U \right) \) with \( k = \text{reaction rate const.} \)
Cartoon showing the deposition of PTFE on the porous silver membrane
Cartoon showing the deposition of PTFE on the porous silver membrane and GDL from SGL (Sigracet 35 DC)
SEM picture of the silver membrane with 0.2µm pore diameter from Sterlitech Inc. (USA)
CV with 1 mV/s at 80°C in 10 M NaOH, O₂
CV's (1 mV/s) from -700 mV to 450 mV vs. Hg/HgO at 80°C in 10 M NaOH, O₂
Current density at -700 mV vs. Hg/HgO and $i_{\text{max}}$ in the reduction peak in function of the pore diameter
Comparison of measured impedance spectra, measured in 10 M NaOH at 80°C, -700 mV vs. Hg/HgO after 60 minutes.
Time dependency of impedance spectra measured at -700 mV vs. Hg/HgO, silver membrane with 1.2 µm pore diameter